

# Scaling of Hall Resistivity in the Mixed State of MgB<sub>2</sub> Films

Huan Yang<sup>1</sup>, Ying Jia<sup>1</sup>, Lei Shan<sup>1</sup>, Chenggang Zhuang<sup>2,3,4</sup>, X. X.

Xi<sup>3,4</sup>, Qi Li<sup>3</sup>, Zikui Liu<sup>4</sup>, Qingrong Feng<sup>2</sup>, and Hai-Hu Wen<sup>1\*</sup>

<sup>1</sup>*National Laboratory for Superconductivity, Institute of Physics and National Laboratory for Condensed Matter Physics, Chinese Academy of Sciences, P.O. Box 603, Beijing 100080, P. R. China*

<sup>2</sup>*Department of Physics, Peking University, Beijing 100871, P. R. China*

<sup>3</sup>*Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA and*

<sup>4</sup>*Department of Materials Science and Engineering,*

*The Pennsylvania State University, University Park, Pennsylvania 16802, USA*

The longitudinal resistivity ( $\rho_{xx}$ ) and transverse resistivity ( $\rho_{xy}$ ) of MgB<sub>2</sub> thin films in the mixed state were studied in detail. We found that the temperature dependencies of  $\rho_{xx}$  and  $\rho_{xy}$  at a fixed magnetic field ( $H$ ) satisfy the scaling law of  $\rho_{xy} = A\rho_{xx}^\beta$ , where the exponent  $\beta$  varies around 2.0 for different fields. In the low field region (below 1 T),  $\beta$  maintains a constant value of 2.0 due to the weak pinning strength of the vortices, mainly from the superfluid of the  $\pi$  band. When  $H > 1$  T,  $\beta$  drops abruptly to its lowest value at about 2 T because of the proliferation of quasiparticles from the  $\pi$ -band and, hence, the motion of the vortices from the superfluid of the  $\sigma$ -band dominates the dissipation. As the field is increased further, the vortex pinning strength is weakened and  $\beta$  increases monotonically towards 2.0 at a high field. All the results presented here are in good agreement with the expectation of the vortex physics of a multi-band superconductor.

PACS numbers: 74.70.Ad, 74.25.Qt, 74.25.Fy

## I. INTRODUCTION

Since the discovery of the binary superconductor MgB<sub>2</sub> in 2001, a number of studies have reported that the properties of its mixed state<sup>1,2,3,4,5,6,7</sup> and normal state<sup>8,9,10</sup> properties have a close relationship with its multiband characteristics. The two three-dimension (3D)  $\pi$  bands and two quasi-two-dimension (quasi-2D)  $\sigma$  bands provide rich physics compared with those materials with a single band.<sup>11</sup> The two sets of bands have different superconducting gaps, i.e., about 7 meV for the  $\sigma$  bands, and about 2 meV for the  $\pi$  bands.<sup>4,12</sup> Moreover, the coherence length of the  $\pi$  bands is much larger than that of the  $\sigma$  bands<sup>1</sup>. Many experiments have demonstrated that the  $\pi$ -band pairing strength is closely related to that of the  $\sigma$ -band and both the interband and intraband scattering play important roles in this multiband system. Owing to the complicated nature of superconductivity in this system, its vortex dynamics may exhibit some interesting or novel features.

For the high critical temperature superconductors (HTSC), the Hall measurements revealed a puzzling scaling relationship between temperature dependence  $\rho_{xx}$  and  $\rho_{xy}$  in the mixed state at a fixed field, i.e.,  $\rho_{xy} = A\rho_{xx}^\beta$ , where the exponent  $\beta$  was observed in the range from 1.5 to 2.0 in YBaCuO<sup>13</sup>, YCaBaCuO<sup>14</sup> and HgBaCaCuO<sup>15</sup>,  $\beta \sim 2$  in BiSrCaCuO<sup>16</sup> and TlBaCaCuO<sup>17</sup>. A number of theories have been proposed in order to explain this scaling law. First,  $\beta$  may be relevant to the scaling parameters of  $I - V$  curves near the vortex-glass transition temperature.<sup>18</sup> Another theoretical model proposed by Vinokur *et al.*<sup>19</sup> suggests that the scaling exponent  $\beta$  should be 2.0 independent of the vortex pinning strength. Later on, Wang *et al.*<sup>20</sup> developed a theory considering both the pinning effect and the ther-

mal fluctuations, and gave the range of  $\beta$  from 1.5 to 2.0 depending on the strength of the vortex pinning. For HTSCs, the experimental results revealed a universal  $\beta$  value about 2.0 at a higher field and a small value at a lower field.

The Hall effect in MgB<sub>2</sub> has been studied in many previous works, and the anomalous Hall effect was found<sup>21</sup>. Jung *et al.* proposed that the field dependence of Hall conductivity  $\sigma_{xy}$  may result from both the vortex motion and the quasiparticles.<sup>22</sup> Similar to the HTSCs, a universal scaling behavior of the Hall resistivity was found in the films, which gives a constant value of  $\beta$  of  $2.0 \pm 0.1$  independent of the magnetic field, the temperature, and the current density, even for the behavior of the field dependence of the two resistivities.<sup>23</sup> In this paper, we present the Hall scaling result in the mixed state, and our results are closely related to the multiband property.

## II. EXPERIMENTS

The MgB<sub>2</sub> thin films used in this work were grown by the hybrid physical-chemical vapor deposition technique<sup>24</sup> on (0001) 6H-SiC substrates. The films were epitaxial and highly  $c$ -axis orientated with a thickness of about 100 nm. There are no impurity peaks in the X-ray diffraction pattern, and the sharp intrinsic peaks show good crystallinity; the  $\phi$  scan (azimuthal scan) indicates the sixfold hexagonal symmetry of the MgB<sub>2</sub> film matching the substrate. The scanning electron microscopy (SEM) image shows no observable grain boundaries in micrometer scale, suggesting a good homogeneity of the films. All samples have a critical temperature of about 40 K with a very sharp resistive transition. A thin layer of gold was sputtered onto the electrode parts of the

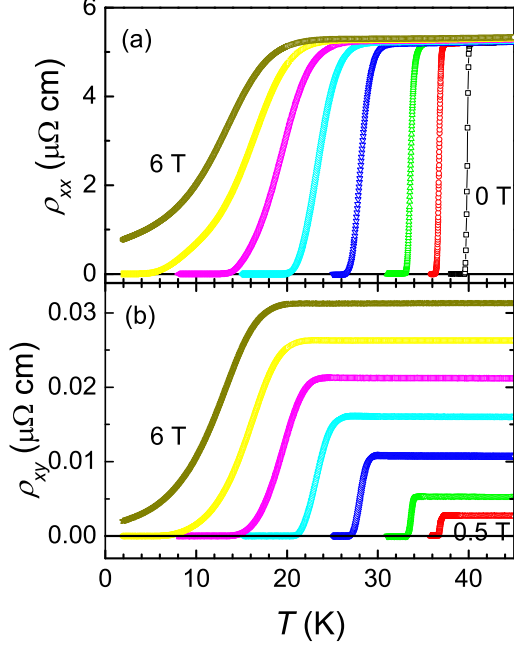


FIG. 1: (color online) Temperature dependence of longitudinal resistivity  $\rho_{xx}$  (a) and transverse resistivity  $\rho_{xy}$  (b). In (a), from right to left, the corresponding fields are: 0, 0.5, 1, 2, 3, 4, 5, 6 T; in (b), from right to left, the corresponding fields are: 0.5, 1, 2, 3, 4, 5 and 6 T.

film, and the contact resistance is smaller than 1  $\Omega$ . In the measurement, bidirectional current mode was applied with a current density of  $10^3$  A/cm<sup>2</sup> in the linear resistivity region<sup>5</sup>. For small current and contact resistance, the self-heating effect of the current in the measurement can be omitted. The temperature dependencies of  $\rho_{xx}$  and  $\rho_{xy}$  were measured at the same time at various magnetic fields applied perpendicular to the  $ab$  plane of the films, and the voltage resolution was about 10 nV. The Hall resistivity  $\rho_{xy}$  was obtained by averaging the results measured in the inverted fields.

### III. RESULTS AND DISCUSSION

Fig. 1 shows the temperature dependence of  $\rho_{xx}$  and  $\rho_{xy}$  of a MgB<sub>2</sub> film measured in various magnetic fields. The transition width of  $\rho_{xx}$  at 0 T is about 0.2 K, and the residual resistivity  $\rho_n$  at 42 K is about 5.3  $\mu\Omega$  cm. The residual resistance ratio  $RRR \equiv \rho(300\text{ K})/\rho(42\text{ K})$  is about 5.1. There is a continuous broadening of transition width as the field increases. At a field larger than 5 T, there is a nonvanishing dissipation at the lowest temperature in the measurement, i.e., 1.9 K, and it has been discussed in detail elsewhere<sup>2,3</sup>.

In Fig. 2, we plot  $\rho_{xy}$  vs  $\rho_{xx}$  at different fields from 0.5 to 6 T in double logarithmic scales. Linear behav-

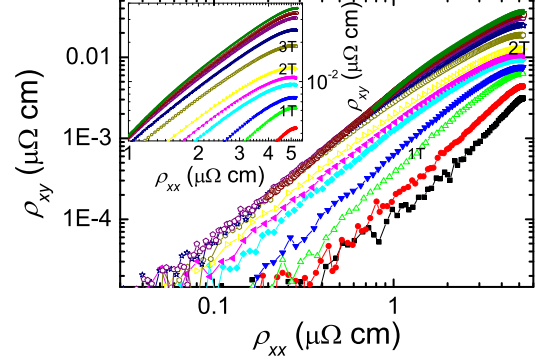


FIG. 2: (color online) Correlation between  $\rho_{xx}$  and  $\rho_{xy}$  measured at magnetic fields of 0.5, 0.7, 1.0, 1.2, 1.5, 1.7, 2.0, 3.0, 4.0, 5.0, 5.5 and 6 T (from bottom to top) in log-log plot. The inset shows the high resistivity section.

ior can be seen clearly in the region of  $\rho_{xx} < 1/4\rho_n$  for almost all fields. Fig. 3 shows the field dependence of the slope  $\beta$  determined by linearly fitting the small resistance regime presented in Fig. 2. As the field increases,  $\beta$  remains constant at 2.0 below 1 T, drops rapidly to a minimal value around 2 T, and increases monotonically at higher fields. Similar results were observed for all studied samples. As an example, the  $\beta$  values obtained from our previous work<sup>3</sup> are also plotted in the figure with open squares. Although the sample studied in the previous work has a lower residual resistivity, the determined slope shows a minimum at an intermediate field of 3 T, which is in good agreement with the data obtained in this work. In Fig. 4, we plot the smoothed differential value of  $\beta$  versus the magnetic field and the normalized resistivity in a 3D plot, and the white line in the surface is the state with the same  $\beta$ . This shows that, when  $\rho_{xx}/\rho_n < 20\%$ , although there is a minimum value at about 2 T,  $\beta$  locates between 1.7 ~ 2.0 depending on the magnetic field similar to the HTSC, indicating the vortex motion. Surprisingly, the value reaches 2.0 and the curves overlap at the high field, starting from 3 T, and the low resistivity region. While, when  $\rho_{xx}/\rho_n$  increases, the  $\beta$  value drops rapidly, showing the joint contribution by both the vortices and the quasiparticles.

In the mixed state of a type II superconductor, the longitudinal and Hall resistivity can be related by the following scaling law<sup>19</sup>:

$$\rho_{xy} = \frac{c^2 \alpha}{\Phi_0 B} \rho_{xx}^2. \quad (1)$$

which is appropriate for any type of vortex motion including flux flow, thermally assisted flux flow (TAFF), and vortex creep in the vortex glass regime. Wang *et al.* proposed another form expressed<sup>20</sup> by:

$$\rho_{xy} = \frac{\beta_0 \rho_{xx}^2}{\Phi_0 B} \{ \eta(1 - \bar{\gamma}) - 2\bar{\gamma}\Gamma(v_L) \}. \quad (2)$$

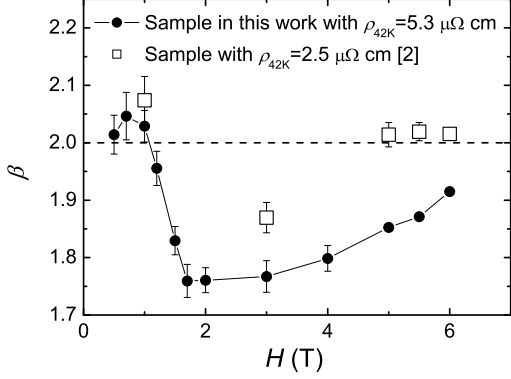


FIG. 3: Magnetic field dependence of the exponent  $\beta$  value by the linear fit to the log-log plot of Hall and transverse resistivities in the small resistivity area of this work (solid circles) and from our previous work<sup>3</sup> (open squares).

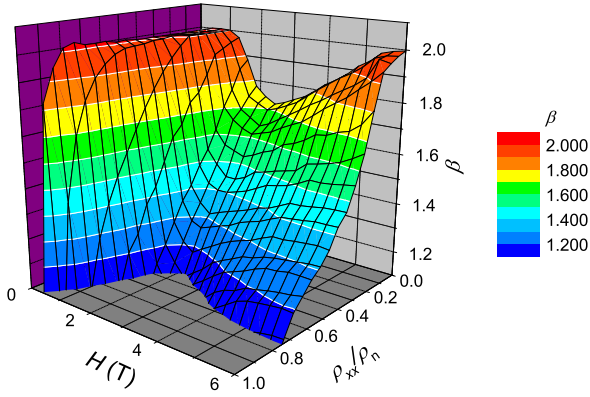


FIG. 4: (color online) 3D plot of the local differential  $\beta$  versus the magnetic fields and normalized longitudinal resistivity. In the region with low resistivity,  $\beta$  locates between 1.7 – 2.0 depending on the field, indicating the vortex motion. At high resistivity region,  $\beta$  drops rapidly, showing the contribution by both the vortices and quasiparticles.

in which  $\beta_0 = \mu_m H_{c2}$ , where  $\mu_m$  is the mobility of the charge carrier and  $H_{c2}$  is the usual upper critical field;  $\Phi_0$  is the quantum of the flux;  $\overline{\gamma} = \gamma(1 - \overline{H}/H_{c2})$  with  $\overline{H}$  the average magnetic field over the core and  $\gamma$  the parameter describing contact force on the surface of core.  $\Gamma(v_L)$  is a positive scale function dependent on the time-average flux-motion velocity  $v_L$  and the Lorentz force. For  $\xi/l \ll 1$  with  $\xi$  the coherence length and  $l$  the mean free path of the charge carrier,  $\gamma \sim 0$  (Nozières-Vinen limit), and for  $\xi/l \geq 1$ ,  $\gamma \sim 1$  (Bardeen-Stephen limit). For MgB<sub>2</sub>, the coherence length of the  $\pi$  band is about 50 nm as derived from the STM measurement<sup>7</sup>, and the calculated value from the upper critical field

of the  $\sigma$  band is about 20 nm. The mean free path of this film is about 25 nm estimated from the resistivity measurement. Hence, the value of  $\gamma$  can be determined to be about 1. In a weak pinning system,  $\Gamma(v_L) \ll \eta \overline{H}/H_{c2}$ , so Eq. 2 becomes  $\rho_{xy} \sim A \rho_{xx}^2$  which is similar to Eq. 1 with  $A$  approximately being a field independent constant. However, for the strong pinning case  $\Gamma(v_L) \gg \eta \overline{H}/H_{c2}$ , and with the rough estimation  $\Gamma(v_L) \sim v_L^{-1/2} \sim \rho_{xx}^{-1/2}$ ,<sup>25</sup> the scaling behavior changes to  $\rho_{xy} \sim A \rho_{xx}^{1.5}$ . With the decrease of pinning strength,  $\beta$  changes continuously from 1.5 to 2.0. As shown in Fig. 3, the continuous increase of  $\beta$  above 2 T indicates that the pinning strength dominates in the scaling law. Moreover, the  $\beta$  value of the cleaner film used in previous work ( $\rho(42K) = 2.45 \mu\Omega \text{ cm}$ ) is clearly larger than that of the dirtier sample here ( $\rho(42K) = 5.3 \mu\Omega \text{ cm}$ ), which also suggests the dominant role of the pinning strength in the current system. For the clean films, at a field above 4 T,  $\beta$  remains at 2.0 with an almost constant coefficient  $A$ . Such a weak pinning regime in the low temperature region at a higher field suggests that the vortex motion associated with the vortex quantum fluctuation is a possible origin of the non-vanishing dissipation in higher fields below  $H_{c2}$ , as addressed in detail elsewhere.<sup>3</sup>

The scaling law of the MgB<sub>2</sub> system observed here seems to be similar to the situation of HTSC at high fields. However, at a field lower than 1 T,  $\beta$  gives a puzzling value of 2.0 independent of the magnetic field. As proposed by Dorsey *et al.*<sup>18</sup>,  $\beta = 1 + \lambda_v/(z + 2 - D)$ , where  $\lambda_v$  is an eigenvalue, while  $z$  and  $D$  are the parameters in the  $I - V$  vortex-glass scaling. In our previous work<sup>5</sup>, we gave the value of  $z$  and  $D$  of the cleaner film mentioned above. The obtained  $z$  value at 3 T is larger than that of small fields from 0.1 to 1 T; while the  $v$  value has an opposite field dependence, which is consistent with the low-field behavior of  $\beta$  presented in this work. All these aspects indicate that the vortex state at a field lower than 1 T is abnormal in MgB<sub>2</sub> compared with a single band superconductor.

MgB<sub>2</sub> is a two-band superconductor, and the  $\pi$ -band superconductivity can be easily destroyed by a small magnetic field. From the theoretical calculation<sup>26</sup> and the point contact spectroscopy,<sup>27</sup> we find that at the magnetic field around 1 T there is an inflexion in the plot of the density of quasiparticles versus magnetic field. In other words, at small fields below 1 T, the  $\pi$ -band pairing is depressed rapidly and enormous quasiparticles are generated. In this regime, the vortices from the superfluid dominated by the  $\pi$  band may play an important role in the conducting property, and the spread of the quasiparticles both outside and inside the vortex cores<sup>7</sup> could reduce the pinning strength of the  $\pi$ -band dominated vortices. At fields higher than 1 T, the pairing strength of the  $\pi$  band is sustained by the coupling to the  $\sigma$  band and the proliferation of quasiparticles increases slowly. Consequently, the  $\sigma$ -band contribution to the vortex behavior becomes increasingly important. The vortices from the superfluid dominated by the  $\sigma$  band may

be easily pinned and, hence, the exponent  $\beta$  drops rapidly to a small value. Finally, when the field increases higher than 2 T, the  $\sigma$ -band dominated vortices play an important role in the transport property, and the situation is very similar to that of HTSC. Furthermore, from the experiment of small-angle neutron scattering<sup>28</sup>, an obvious vortex structure change from 0.5 to 0.9 T was observed in the single crystal sample, which may imply that the  $\pi$ -band vortex lattice changes to the  $\sigma$ -band vortex lattice, though this phenomenon still needs further consideration.

#### IV. CONCLUSIONS

We have presented the scaling law between the Hall and longitudinal resistivity at fixed magnetic fields for samples with different pinning strengths. The scaling exponent  $\beta$  equals 2.0 at low fields (below 1 T), which may be associated with the weak-pinning  $\pi$ -band dominated vortices. When the field reaches 2 T, the  $\sigma$ -band

dominated vortices play an important role in the conductivity, and the stronger pinning strength reduces the value of  $\beta$ . By further increasing the field, the vortex pinning is weakened continuously and  $\beta$  increases and finally approaches 2.0 in the low-resistivity region again. This is another proof that the multiband property is very important in the mixed state of MgB<sub>2</sub>.

#### Acknowledgments

This work is supported by the National Science Foundation of China, the Ministry of Science and Technology of China (973 project: 2006CB601000 and 2006CB921802), and the Knowledge Innovation Project of the Chinese Academy of Sciences (ITSNEM). The work at Penn State is supported by NSF under Grants Nos. DMR-0306746 (X.X.X.), DMR-0405502 (Q.L.), and DMR-0514592 (Z.K.L. and X.X.X.), and by ONR under grant No. N00014-00-1-0294 (X.X.X.).

- 
- \* Electronic address: hhwen@aphy.iphy.ac.cn
- <sup>1</sup> A. E. Koshelev, and A. A. Golubov, Phys. Rev. Lett. **92**, 107008 (2004); A. A. Golubov, and A. E. Koshelev, Phys. Rev. B **68**, 104503 (2003).
  - <sup>2</sup> H. H. Wen, S. L. Li, Z. W. Zhao, Y. M. Ni, Z. A. Ren, G. C. Che, H. P. Yang, Z. Y. Liu, and Z. X. Zhao, Chin. Phys. Lett. **18**, 816 (2001); H. H. Wen, S. L. Li, Z. W. Zhao, H. Jin, Y. M. Ni, W. N. Kang, H. J. Kim, E. M. Choi, and S. I. Lee Phys. Rev. B **64**, 134505 (2001).
  - <sup>3</sup> Y. Jia, H. Yang, Y. Huang, L. Shan, C. Ren, C. G. Zhuang, Y. Cui, Q. Li, Z. K. Liu, X. X. Xi, and H. H. Wen, arXiv:cond-mat/0703637 (unpublished) (2007).
  - <sup>4</sup> H. J. Choi, D. Roundy, H. Sun, M. L. Cohen, and S. G. Louie, Nature (London) **418**, 758 (2002).
  - <sup>5</sup> H. Yang, Y. Jia, L. Shan, Y. Z. Zhang, H. H. Wen, C. G. Zhuang, Z. K. Liu, Q. Li, Y. Cui, and X. X. Xi, Phys. Rev. B **76**, 134513 (2007).
  - <sup>6</sup> R. S. Gonnelli, D. Daghero, G. A. Ummarino, V. A. Stepanov, J. Jun, S. M. Kazakov, and J. Karpinski, Phys. Rev. Lett. **89**, 247004 (2002).
  - <sup>7</sup> M. R. Eskildsen, M. Kugler, S. Tanaka, J. Jun, S. M. Kazakov, J. Karpinski, and Ø. Fischer, Phys. Rev. Lett. **89**, 187003 (2002).
  - <sup>8</sup> I. Pallecchi, V. Ferrando, E. Galleani D'Agliano, D. Marré, M. Monni, M. Putti, C. Tarantini, F. Gatti, H. U. Aebersold, E. Lehmann, X. X. Xi, E. G. Haanappel, and C. Ferdeghini, Phys. Rev. B **72**, 184512 (2005).
  - <sup>9</sup> Qi Li, B. T. Liu, Y. F. Hu, J. Chen, H. Gao, L. Shan, H. H. Wen, A. V. Pogrebnnyakov, J. M. Redwing, and X. X. Xi, Phys. Rev. Lett. **96**, 167003 (2006).
  - <sup>10</sup> I. I. Mazin, O. K. Andersen, O. Jepsen, O. V. Dolgov, J. Kortus, A. A. Golubov, A. B. Kuz'menko, and D. van der Marel, Phys. Rev. Lett. **89**, 107002 (2002).
  - <sup>11</sup> P. C. Canfield, and G. Crabtree, Phys. Today **56**, 34 (2003).
  - <sup>12</sup> M. Iavarone, G. Karapetrov, A. E. Koshelev, W. K. Kwok, G. W. Crabtree, D. G. Hinks, W. N. Kang, E. M. Choi, H. J. Kim, H. J. Kim, and S. I. Lee, Phys. Rev. Lett. **89**, 187002 (2002).
  - <sup>13</sup> J. Luo, T. P. Orlando, J. M. Graybeal, X. D. Wu, and R. Muenchausen Phys. Rev. Lett. **68**, 690 (1992); W. N. Kang, D. H. Kim, S. Y. Shim, J. H. Park, T. S. Hahn, S. S. Choi, W. C. Lee, J. D. Hettinger, K. E. Gray, and B. Glagola, Phys. Rev. Lett. **76**, 2993 (1996).
  - <sup>14</sup> Z. Wang, Y. Z. Zhang, X. F. Lu, H. Gao, L. Shan, and H. H. Wen, Physica C **422**, 41 (2005).
  - <sup>15</sup> W. N. Kang, S. H. Yun, J. Z. Wu, and D. H. Kim, Phys. Rev. B **55**, 621 (1997).
  - <sup>16</sup> A. V. Samoilov, Phys. Rev. Lett. **71**, 617 (1993).
  - <sup>17</sup> A. V. Samoilov, Z. G. Ivanov, and L. G. Johansson, Phys. Rev. B **49**, 3667 (1994).
  - <sup>18</sup> A. T. Dorsey, and M. P. A. Fisher, Phys. Rev. Lett. **68**, 694 (1992).
  - <sup>19</sup> V. M. Vinokur, V. B. Geshkenbein, M. V. Feigel'man, and G. Blatter, Phys. Rev. Lett. **71**, 1242 (1993).
  - <sup>20</sup> Z. D. Wang, J. M. Dong, and C. S. Ting, Phys. Rev. Lett. **72**, 3875 (1994).
  - <sup>21</sup> R. Jin, M. Paranthaman, H. Y. Zhai, H. M. Christen, D. K. Christen, and D. Mandrus, Phys. Rev. B **64**, 220506(R) (2001).
  - <sup>22</sup> S. G. Jung, W. K. Seong, J. Y. Huh, T. G. Lee, W. N. Kang, E. M. Choi, H. J. Kim, and S. I. Lee, Supercond. Sci. Technol. **20**, 129 (2007).
  - <sup>23</sup> W. N. Kang, H. J. Kim, E. M. Choi, H. J. Kim, K. H. P. Kim, and S. I. Lee, Phys. Rev. B **65**, 184520 (2002).
  - <sup>24</sup> X. H. Zeng, A. J. Pogrebnnyakov, A. Kotcharov, J. E. Jones, X. X. Xi, E. M. Lysczek, J. M. Redwing, S. Y. Xu, Q. Li, J. Lettieri, D. G. Schlom, W. Tian, X. Q. Pan, and Z. K. Liu, Nature Mater. **1**, 1 (2002).
  - <sup>25</sup> V. M. Vinokur, V. B. Geshkenbein, M. V. Feigelman, and G. Blatter, Phys. Rev. Lett. **71**, 1242 (1993).
  - <sup>26</sup> A. E. Koshelev, and A. A. Golubov, Phys. Rev. Lett. **90**,

- 177002 (2003).
- <sup>27</sup> R. S. Gonnelli, D. Daghero, A. Calzolari, G. A. Ummarino, V. Dellarocca, V. A. Stepanov, J. Jun, S. M. Kazakov, and J. Karpinski, Phys. Rev. B **69**, 100504(R) (2004); Y. Bugoslavsky, Y. Miyoshi, G. K. Perkins, A. D. Caplin, L. F. Cohen, A. V. Pogrebnyakov, and X. X. Xi, Phys. Rev. B **72** 224506 (2005); A. Kohen, F. Giubileo, Th. Proslier, F. Bobba, A. M. Cucolo, W. Sacks, Y. Noat, A. Troianovski, and D. Roditchev, Eur. Phys. J. B **57**, 21 (2007).
- <sup>28</sup> R. Cubitt, M. R. Eskildsen, C. D. Dewhurst, J. Jun, S. M. Kazakov, J. Karpinski, Phys. Rev. Lett. **91**, 047002 (2003).